Mountain Bike Suspension Capstone

Final Report

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DISCLAIMER

This report was prepared by students as part of a university course requirement. While considerable effort has been put into the project, it is not the work of licensed engineers and has not undergone the extensive verification that is common in the profession. The information, data, conclusions, and content of this report should not be relied on or utilized without thorough, independent testing and verification. University faculty members may have been associated with this project as advisors, sponsors, or course instructors, but as such they are not responsible for the accuracy of results or conclusions.

EXECUTIVE SUMMARY

The initial project description consisted of the team creating a mathematical model to allow users to understand when to adjust the rebound settings based upon trial ratings. With this, the team used two initial platforms of MATLAB and Excel to find the best results for the riders to understand. With Excel being a user-friendly platform that allows for multiple equations to be inserted, the team decided that Excel would be the best course of action. The mathematical model has a main tab that allows the user to insert values into the spreadsheet such as bike mass, fork and spring stiffness, and travel. These inserted components, along with the calculated values such as damping coefficients, speed and frequency of the shock, mass, and weight bias are then used to create a displacement vs. time chart. This chart displays the overall calculated values for the front fork and rear spring to determine if they are either under or over damped. Along with this, a weight bias and shock setup tab were created that correlate to the main initial setup tab. The weight bias portion was measured based upon the rider's weight at the anticipated grade percentage for both ascending and descending. The purpose of determining the weight bias allows the rider to understand which setting of rebound to use based upon the grade at which the trial is rated for. After this tab of the mathematical model was complete, a shock setup tab was made to allow the rider to input the rider weight, ascending or descending, and the type of terrain. The overall objective of the section will display to the rider of how many clicks of rebound will be needed to ride on that specific terrain setting. This final section completed the team's goal of the mathematical model by allowing a rider to insert values to determine the proper amount of rebound used based upon the grade and initial as previously stated. After the team completed the mathematical model, an additional step was taken by both designing and creating a device that can change the suspension settings on the fly while riding. While this idea was not initially needed in the project description, the team wanted to take the project one step further and tie the mathematical model into a physical device that riders could use on the trail. With that said, the D4P pillars were used to start the initial design of creating a morph matrix of potential designs. After a final concept was selected, the design team focused on a cable concept where a lever board would be used. As the lever increases in the slot, a higher tension would result in the cable to move the cap that is adjusted to the rebound. This final design can be found in the implementation section where all the components can be found. All components of the final design were produced through SolidWorks, and where then 3D printed through the NAU maker lab. Additionally, the project bike that was used for testing was provided through Niner bikes. Niner bikes generously provided a great amount of feedback toward the team's approach for both the mathematical model and the design process. With this, Niner also informed the team about the linear potentiometers that measure the speed vs. position. This system allows the team to determine if the system is balanced. The linear potentiometers mount to both the front fork and the rear floater shock and are wirelessly used through an app to collect the data. The data that was pulled from various trial ratings were then compiled into scatter plot graphs to indicate the linear regressions from position in relation to the speed compression. All in all, the following capstone team completed all the required targets from both the client and instructor and took an additional step to create a physical device and relate it to the mathematical model.

ACKNOWLEDGEMENTS

We would like to thank our professor, Dr. Trevas, and our client, Brandon Lurie, for all their guidance and support throughout the last two semesters. We would also like to thank George and Peter from Niner bikes for sending us a bike to test on and build our device, as well as their mentorship. Robert and Michael from Motion Instruments were helpful in getting us a linear potentiometer testing system without any bureaucratic delays.

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1 BACKGROUND

1.1 Introduction

Mountain biking is a sport, hobby, and even a lifestyle for those working in the industry. A key component on modern day mountain bikes is the suspension. For some mountain bikes there are only front suspension forks on the bikes giving them the name "hard tail". However, the most versatile suspension platform is the full suspension bike. A full suspension mountain bike has a front suspension fork and a rear shock which is known to provide comfort and stability when going over harsh terrain, increased traction, and be more capable than their hard tail counterparts. Suspension on mountain bikes is meant to be taken over terrains ranging from a smooth surface to drop offs.

The best method to ensure the suspension can handle these differences in terrain is by utilizing air shocks rather than spring, or coil shocks. Air shocks have various types of adjustments that allow them to be adjusted based on the riding conditions and rider's preference. This ability to adjust the way the bike rides, as well as weight advantage, have made air suspension the best choice for most bikes. The front fork can be locked out with a level to make the front completely rigid; the rebound can be adjusted to dictate how fast the fork reacts to bumps, and air can be added based on the weight of the rider. Rear shocks can also lock out with the damping adjustment, rebound can be increased or decreased, and air and volume spacers can be added to ensure the shock is setup properly. With all the various adjustments, adjusting suspensions can be daunting for the average consumer. Most of the time bike shops will help the rider setup the bike that will handle most terrain decent. To create the best riding bike for differing terrains, adjustments need to be made before each ride if the riding varies. For example, suspensions that are setup for jumps and drop offs will need to be adjusted if the same bike is taken to a flatter terrain with more small bumps. Without knowing exactly how each adjustment affects the bike, the average consumer could feel overwhelmed or helpless when it comes to adjusting their suspension. This project aims to define what a well setup suspension system is along with a mathematical model to follow. This then will lead the team to use a mountain bike and rear shock to apply real-world testing to validate the mathematical model. Once the mathematical model has been validated, a device will be made to help riders adjust their suspension easier. By creating a way for mountain bike suspension to be more easily adjustable, all consumers will be able to ensure their bike is perfectly dialed in no matter what terrain they are riding.

1.2 Project Description

Following is the original project description provided by the sponsor*:*

"Mountain bike suspensions are configurable based on the rider and the types of terrain where the bike is expected to be used. This project involves reverse engineering suspension systems for a mountain bike and analyzing how it would perform for varying terrains for a subset of riders using mathematical modeling. Reverse engineering a design is a common technique used to understand choices/trade-offs that a designer had to make to when developing their product as well as a starting point for creating something similar yet improved or optimized for a subset of customers. The final report should include an easy-touse guide for selecting suspension parameters based on rider details and the range of terrains that the rider is expected to use the bike."

The original project description involved creating a database as a reference that mountain bikers can use to fine tune their suspension. After some initial discussions with our client, Brandon Lurie, the team decided this idea would be less feasible with either too much variance in suspension platforms or products on the market that already accomplish the same goal. The only components our client wanted to keep are the idea of a mathematical model due to his knowledge in creating these models throughout his career. The client was still very open to ideas and provided the team full control of the future of the project. With this, understanding how mountain bike suspension works is pivotal to this project and the team decided

on attempting to build a physical device for this project. With an updated goal in mind, the team now aims to create a mathematical model to help define a well damped suspension system and use physical devices to test the viability of the mathematical model. Once the mathematical model has been created and validated, a device will be made to allow for all riders to adjust their suspension easier.

2 REQUIREMENTS

This Senior Design Project is the capstone requirement for Mechanical Engineering majors where the team has chosen the Reverse Engineer Mountain Bike Suspension project. The team has further clarified the project to designing a system that allows for increased dynamic adjustability for a full suspension mountain bike. To accomplish this the team will utilize semester one to create a mathematical model to model and predict ideal suspension settings based on rider, bike, and terrain characteristics. From the code, a physical prototype mechanism will be developed during semester two and fitted to a to-bedetermined test bike.

2.1 Customer Requirements (CRs)

The project sponsors W.L. Gore and NAU engineering, represented by Brandon Lurie and Dr. David Trevas respectively determined the baseline for customer requirements during this initial stage of project development. The team and clients together determined five customer requirements being: Base research on current mathematical models, perform extensive research on bike suspension systems, ensure the average user can utilize design, incorporate Solid Works and Excel models, and perform validation testing. These customer requirements focus on the team pursuing an analytical approach to the first semester of the design project. There is a large emphasis on truly understanding the mechanisms and properties of current mountain bike suspension in order to move forward with designing a system to work alongside and optimize suspension. This led to the team all taking on individual analysis of suspension systems such as shock quantification or linkage analysis. Each customer requirement was given a weight on a scale one to five. The Solid Works and Excel models are given a five, validation testing a four, bike suspension research a three, and user friendly and current systems research was given a two. These weights were assigned based on their correlation to gaining better understanding of the suspension system and how well they progressed the team's understanding.

The customer requirements from the preliminary report translated into similar requirements for our final proposal. The major change here is the way in which we are taking on the mathematical model. It was determined that utilizing Excel instead of MATLAB lends well to how our model has been designed. Due to this the mathematical model customer requirement is now specified as using Excel to formulate our mathematical model moving forward.

2.2 Engineering Requirements (ERs)

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2.3 Functional Decomposition

Since this project primarily focuses on a mathematical model for the current state of the project, the team could not create some diagrams that were more concept based. The Black Box Model seen below helps the team visualize how the system is currently affected by various inputs. Also, a simple process diagram helps the team maintain vision throughout the remainder of the project.

2.3.1 Black Box Model

A Black Box Model, as seen in figure 2, aims to show the inputs to the proposed system and the outputs of this system in the form of materials, energies, and signals. For this project the goal is to create a device to adjust mountain bike, MTB, suspension on the trail that is easy to understand. The material inputs will be the weight of the bike and the rider, with the rider and bike weight also being the material outputs. Material inputs can be controlled, or known, which allows the team to adjust the system based on the weight of both the person and the bike. Terrain impacts are the energy input, which causes an acceleration of the shock. This reaction to the terrain is something the team wants to be able to adjust with the device. Current adjustments on modern mountain bikes can sometimes be difficult to understand, leading to the output of a modified, or easier, way to adjust mountain bike suspension.

Figure 1: Black Box Model

2.3.2 Functional Model/Work-Process Diagram/Hierarchical Task Analysis

Since the preliminary report, the team has developed a process diagram that outlines the necessary major tasks throughout the project in chronological order. The process diagram below helps the team maintain vision and guides all future efforts. This models our desired progression for the project and the team is currently finishing the first box and looking to move towards the second process as the semester concludes.

Figure 2: Process Diagram [1]

From the first bubble in the diagram above, the team has already quantified terrain types using the International Mountain Bicycling Association (IMBA) trail rating system, which may be viewed in Appendix A. This process was detailed in the preliminary report. The team has since built a mathematical model in Microsoft Excel, which will be explained in section 5. Initially, the team used MATLAB, but quickly discovered that Excel provides a more streamlined platform for spring mass dashpot calculations.

As for the other two bubbles, the team hopes to begin empirical testing over winter break. Relating math model outputs to actual bike suspension adjustments should validate the model itself. Once physical optimization data are acquired, the team will begin concept generation for the remote adjustment device. The final design should feature optimization settings for the five IMBA trail difficulties. The device should also adjust front and rear suspension independently. These future tasks should easily fill next semester. The team will need to use this process diagram to maintain vision and stay on track as our project translates into physical testing and prototyping.

2.4 House of Quality (HoQ)

The House of Quality or Quality Functional Deployment (QFD) for this project ranks a small list of customer needs against the few engineering requirements developed thus far. It is subject to change and further development as the team progresses from a mathematical model to a physical mountain bike suspension adjustment device. The schematic may be viewed in Figure 3 below.

Durability													
Compact Design to Fit on Handlebars													
Minimize Weight Addition					$+$								
Compatible with Test Bike		$+$		$+$	÷	٠							
Validate Mathmatical Model with Other Models			$+$										
Spring and Damping Rate Critical for all Cases		$^{+}$	$+$	$^{+}$									
PHASE I QFD													
						Engineering Requirements (How)					Now vs Now What		
Customer Needs (What)	Customer Weights	Spring and Damping Rate Critical for all Cases	Validate Mathmatical Model with Other Model	Compatible with Test Bike	Minimize Weight Addition	Compact Design to Fit on Handlebars	Durability	1 Poor	\sim	3 Acceptable	4		5 Excellent
Base Research on Current Mathmatical Models	$\overline{2}$	9	$\overline{9}$							AB			
Extensive Research on Bike Suspension System	$\overline{\mathbf{3}}$			$\overline{\mathbf{3}}$			\mathbf{I}			Λ	$\, {\bf B}$		
Ensure the Average User can utilize Design	$\overline{2}$	\mathbf{I}		$\overline{9}$	\mathbf{I}	3	9					AB	
Incorporate Solid Works and MATLAB	$\overline{\mathbf{5}}$		9	$\overline{3}$	$\overline{\mathbf{3}}$	$\overline{3}$					AB		
Validation Testing	4	$\overline{\mathbf{3}}$		$\overline{9}$							AB		
	Absolute Technical Importance (ATI)	32	63	78	$\overline{11}$	\overline{z}	\overline{z}					Benchmarked Products	
	Relative Technical Importance (RTI)	14%	27%	34%	7%	9%	9%		A		Fox Live Valve		
	Unit of Measure	Y/S	Y/S	Y/S	Kg	cm^3	lbf		\overline{B}			RockShox Brain	

Figure 3: House of Quality

Engineering Requirements with the highest absolute and relative technical importance include ensuring a critical damping rate (ATI 32, RTI 14%), validating the mathematical model with empirical testing of a physical bike (ATI 63, RTI 27%), and making sure the suspension adjustment device developed by the team is compatible with the bike tested (ATI 78, RTI 34%). The critical damping rate will vary as terrain and trail difficulty varies, and therefore is unknown at this point. Hopefully the team will confirm critical damping rates calculated in the mathematical model when physically testing the mountain bike, validating the model. At the end of the project, the team should have a suspension adjustment device designed specifically for the bike tested. Any rider should be able to adjust the suspension to the desired terrain setting based on suggestions from the previously developed mathematical model. This QFD helps the team realize the most important engineering requirements in order to reach the end goal.

2.5 Standards, Codes, and Regulations

ASTM has standard for mountain bikes in the following paragraph:

"This specification establishes the performance requirements and associated test methods for qualifying designs of suspension and nonsuspension production forks employed on bicycles that are intended for use in Condition 3 topography. This kind of condition pertains to rough trails, rough unpaved roads, and rough technical areas and unimproved trails, wherein contact with the irregular terrain and momentary loss of tire

contact with the ground may occur during usage. The forks shall go through compression load, bending load, impact resistance, and fatigue tests. Models that fail to meet the specified test requirements shall be rejected." [2]

There are no specific regulations that abstracts the usage of new types of damper fluid, or how much nitrogen is used in the shock, or the usage of any material specifically. As can be summarizes from the ASTM paragraph, if the part works properly and effectively works as intended, it won't get rejected [2]. Also, flammable gas like nitrogen is to be used, manufacturers are required to seal it preventing any leakage [3]. This will not affect our project in any major ways since we are not manufacturing our own suspension, instead we are going to use an existing suspension that already met all the requirements.

These are not set-in-stone by any means, they are only frequently used in bikes. This results in an extremely wide range of possibilities. The part that proved to provide the most possibilities was the damper fluid. There are more than 100 known damping fluids that manufacturers could use in bikes dampers [6].

3 DESIGN SPACE RESEARCH

[Provide the sections of the Final Proposal from ME 476C here, mitigating any and all issues from the last report.]

3.1 Literature Review

For the first source, we found a website that helps with teaching us how to do stress analysis in SolidWorks [1]. Because the university provided this program free of charge, this website was a huge help in giving us a better understanding of what fixed geometries we should use in the analysis as well as hot to compose a stress analysis study. For the second source, this research paper included a research testing vibration in individual bike parts, when going on multiple trail types [2]. This helped the team classify which part would need more attention than others when minimizing the vibrations on the bike rider.

The third researched source was a paper analyzing Magnetorheological (MR) fluids and their applications in suspensions [3]. MR fluids proved to be possessing properties that would help us in shock minimization, like fast response time and field-dependent yield stress. The paper even compared them with passive suspensions and the results showed that MR fluids performed better. However, due to our project focusing on maximizing the performance of what the costumer already has we ended up not adding them. The fourth used source was a paper researching wheels with spring systems that absorbed shock [4]. Unfortunately, the shock absorption comes at the price of requiring more power from the rider. The shocks we faced did were not severe which is why we decided that the wheels are not required.

The next paper talked about what effects spring shapes had on the spring's properties without changing the material used [5]. This paper helped shine a light on more improvements we can make on the device since we ended up requiring the use of springs. The source after that was a paper that compared rigid shock to mountain bike shocks in three different settings [6]. The results showed overwhelming advantage in on the side of the mountain bike shocks. The softer fork setting resulted in reduced impact acceleration on the rider and the middle setting showed the best damping for impact acceleration. This was used to improve the comfort the mathematical model settings would provide. The last source was a physical system and dynamics book that equipped us with knowledge on the right use of Laplace Transform [7]. Laplace Transform was used to solve multiple second order differential equations. This was extremely helpful in developing the mathematical model's calculations.

3.2 Benchmarking

Benchmarking is a vital step to the design process is engineering. It is described as the process of measuring designs and/or processes from other designers or companies in order to gain insight on what is already on the market. In doing this, engineers and designers are able to find ways to help smooth out the process of designing and producing products. For the team, benchmarking was an opportunity to see what was already on the market and if those products fit our project statement.

3.2.1 System Level Benchmarking

For Benchmarking, the team decided to look at designs that resembled the idea for a physical device that could adjust suspension while on the trail. This would give the team a good idea of what is already on the market and what could be improved on. These existing designs are the primary examples of similar products that can currently be found on the market.

3.2.1.1 Existing Design #1: Specialized Brain

This product was designed specifically for cross country mountain bikes by the company Specialized. The Specialized Brain uses the inertia of a bump to open up a valve and let oil flow in the shock, letting it compress. When the biker is stationary and/or peddling on a smooth surface, the valve stays closed so the biker is not wasting energy through the shock compression. However, whenever the biker hits a bump on the trail, the shock will compress as normal because of the opening of the valve. This product covers our want for an easy-to-use product but doesn't necessarily cover all of our requirements. [8]

3.2.1.2 Existing Design #2: Fox Live Valve

The Fox Live Valve is similar to the Specialized Brain in that it automatically senses the terrain to open and close a valve in the shock to control the rebound except it uses an electronic controller to control flow rather than the inertia valve used in the Specialized Brain. This electronic system makes it possible for the user to choose what kind of ride they want to have using the included app controller. This system comes closer to what we would like to create because you have the option of what terrain you will be riding on. [9]

3.2.1.3 Existing Design #3: Quarq Shock Wiz

The Quarq Shock Wiz is a device that relays information from the front and rear suspensions on a mountain bike. The information given gives riders advice on the best ways to tune their suspension to fit their needs on the trail. This is kind of similar to the Fox Live Valve except it only can give suggestions and not actually adjust the suspension accordingly. This may seem like a drawback, but this is probably the most viable option for people because it can be fitted on most suspensions, not just a single brand. [10]

4 CONCEPT GENERATION

The concept generation for the design portion of this capstone project consisted of following the D4-P pillars to ensure that the final design was well thought out by each team member. To start, each design team member contributed a total of ten sketches that relate to the five initial design categories. These sketches were then used to generate a morph matrix to display all the potential ideas. From there, the team then used a Pugh chart to rank the best design in that category. The Pugh chart is beneficial because the team discussed all the pros and cons of each sketch, and then ranked them based on the engineering requirements. As seen below, figure 4 shows the five initial design categories that the team used.

Figure 4: Five Initial Design Categories

As seen in the tables below, a total of thirty-six sketches were created to find solutions for the final product. With that said, almost every design concept that was created was tied into the final design in some type of way.

Table 2 : Morph Matrix

After creating the morph matrix, the team then used a template as seen in table below to determine the best ranked deigns per each category. As seen in the table, the requirements on the left-hand side in red were used to determine the rankings for each design. Aa datum design was also determined to compare the results between multiple designs.

After the final designs were selected based upon the Pugh charts, the team then used SolidWorks to begin the manufacturing process. The final designs that were designed through the concept generation process will be seen in the next section, for all the components will be discussed thoroughly.

5 DESIGN SELECTED – First Semester

The final mathematical model will include input variables such as terrain, rider size and various shock dimensions, then output the behavior of the shock which will give the team insight on what adjustments to recommend to the rider. Microsoft excel will be used to create the mathematical model with a sheet for each level of terrain. An analysis of the spring stiffness will be included in this section since air shocks do not behave as linear springs. Once the spring stiffness is estimated using the linear correlation of the data, the data will be compiled and analyzed. While the mathematical model is finished, the team still needs to analyze how outputs from the model can be converted into suspension adjustments.

Design Description: **Spring Stiffness**

Since most modern mountain bikes use air shocks instead of a spring or coil shock, a model needed to be generated to include various parameters of the shock's dimensions and the amount of pressure inside the shock. The equation (equation 1 below) used to model the air shocks on mountain bikes spans from U.S. Patent No. 4,629,170, which is then discussed in an analysis in European Patent No. 0285726B1.

$$
K = \frac{nPA_e^2}{V} + \frac{P_g dA_e}{dH}
$$
 (1)

In this equation, K is the stiffness of the spring, P is the absolute internal pressure, Pg is the gauge pressure, Ae is the effective area acted on by air pressure, V is the air volume, n is the ratio of specific heats, and is the change in effective area with spring height. The absolute internal pressure will vary by the rider's weight and the type of shock. Area and volume will also be dependent on the specific shock and can be adjusted for any shock if the dimensions are known. A graph of the spring stiffness versus the length of the shock can be seen below in figure 5.

Figure 5: Spring stiffness through length of shock stroke

This graph shows how the shock stiffness begins to decrease as the rider gets into the travel. For the analysis of the suspension, the team is using the spring-mass dashpot (a differential equation) that needs to be solved numerically. To help simplify the analysis, a linear trend line will be utilized, and stiffness values will be found using the equation of the line. Spring stiffness will vary based on the difficulty of the terrain. For example, a rider on an easier trail will not experience as many, or as large, of bumps meaning they will not use as much of their travel. Versus someone who is going off of a 5 foot drop off and

bottoming out their shock using the entire length of the shock. This will be incorporated in the final mathematical below.

Stress Analysis

The following MATLAB code indicates the stress levels on the crankarms and the seat post (sitting and standing while riding). A SolidWorks Simulation was conducted with a pre-designed mountain bike from *Grab CAD* to determine the Von Mises stresses (2) throughout the geometry of the bike. As seen in the SolidWorks figure (6), the color coordinated bike frame indicates where small, medium, and high stress levels occur. The MATLAB code (7) is an additional source to back up the original trial on the SolidWorks simulation. The results show that when sitting on the seat at an applied load of 1,500N will result in a Von Mises stress of 2.6Mpa and the crankarm will have a Von Mises stress of 3.4Mpa. The MATLAB code is the skeleton of the SolidWorks process, for this information is not seen in the simulation. This allows the team to better understand the mathematical modeling with the stress analysis when analyzing stress throughout the bike frame. Overall, between both software's, the same data was found, allowing the team to know the final Von Mises stresses throughout the geometry of the bike.

$$
\sigma_v = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}
$$

Equation 2: Von Mises Stress

Figure 6: SolidWorks Von Mises Stress

Figure 7: MATLAB Code for stress on Crank Arms

Mathematical Model

For the first semester, the desired product was a mathematical model that outputs suspension reactions to various terrain types. Initially, the team thought MATLAB was the best platform, but Microsoft Excel turned out to be more streamlined. In figure 8 below, all inputs are applied to the front air fork and rear air spring independently. This is important because the components behave differently due to frame geometry and front/rear mass bias. The green cells are user defined, and yellow cells calculate initial values based on the user defined inputs. Bike mass and rider mass are transformed into front and rear mass bias. Spring stiffness comes from the stiffness analysis above, and the damper value represents the mass flow rate of oil inside the damper. The leverage ratio influences rear shock characteristics relative to the front fork, therefore, initial shock speed will always be less for the rear shock. Stroke length is also less than half that of the front fork, causing the displacement profile to look shorter.

Figure 8: Math Model Inputs [7]

These inputs run through an if statement that determines the damped situation (underdamped, critically damped, or overdamped) and then uses the appropriate equation to output a graph of displacement vs time for front and rear suspension

independently (Figure 9). The oscillations both near the baseline within one second reliably, and the interval between data points is 0.01s. Given the inputs above, front fork displacement nearly reaches maximum stroke length, and the rear shock reaches 80% of its stroke length. Critically damped and slightly underdamped situations are desirable for most terrains. Overdamped situations do not allow the shock to sufficiently rebound before the next impact, creating a harsh sensation. The equations behind critically damped and underdamped situations are listed respectively below.

$$
x(t) = [x_0 + (v_0 + \omega_n x_0)t]e^{-\omega_n t}
$$
 Equation 3

$$
x(t)\hspace{-0.1cm}=e^{-\varsigma\omega_n t}\left[x_0cos\omega_dt+\frac{v_0+\varsigma\omega_nx_0}{\omega_d}sin\omega_dt\right]
$$

Equation 4

$$
\omega_d = \omega_n \sqrt{1 - \zeta^2}
$$
 Equation 5

Damped natural frequency ω d is a function of natural frequency ω n and ς in the underdamped equation.

Figure 9: Math Model Outputs [7]

Final Design

The final device design comprises of a five-part 3D printed assembly. We utilized the engineering design process to develop our mathematical model and utilized the model results to properly dimension a mechanical based device. The complete device is comprised of of a top-tube mounted shifter plate, shift wire running to a shock mounted body piece, suspension dial caps, and a spring to keep the wire in tension throughout all setting positions. There is a total of five setting positions that correlate to the five main setting adjustments seen in our model. This shifter assembly is shown in the figure below.

Figure 10: Shifter Assembly

This shifter assembly leads into our rear shock device mount. This component is seen in the figure below and guides the wires around the suspension adjustment caps and into the tension spring.

Figure 11: Rear Shock Bracket

Implementation Plan

This mathematical model will be implemented with real world testing of the bike and studying how different adjustments affect the ride. Once some experimental data is collected the team will be able

to analyze the effectiveness of the model and then optimize the suspension for all terrains. To optimize the suspension settings, the team is looking for a company or local bike shop to allocate a bike for the team to test. If not, the Santa Cruz Heckler used for the sag test will be adequate. During these tests, the team will implement an Arduino with proximity sensors to measure how the bike suspension behaves on various terrains. Ideally, the team will build an alpha prototype (and possibly a beta) of the final adjustment device design. Materials and costs cannot be determined at this time, but the device should stay well within budget, especially if the team uses Dylan's Santa Cruz as the test bike.

6 IMPLEMENTATION – Second Semester

The team focused on updates and optimization of both the mathematical model and the design. The team utilized testing and engineering analysis to arrive at our final optimized systems.

6.1 Design Changes in Second Semester

This semester the team updated the math model to work with our new test bike, then calibrated and validated it with testing. Changes were made to the model and testing apparatus. They will be detailed below. Also, the team designed a device to change rear suspension settings remotely while out on the trail. This device passed through a few design iterations before the end of the semester.

6.1.1 Design Iteration 1: Change in Mathematical Model discussion

Last semester, the math model utilized mass, suspension travel, and leverage ratio inputs for Dylan's 2016 Santa Cruz Heckler. It also outputted suspension reactions only for level terrain. The more difficult trail types can have grades in excess of 15%, so the team built in weight bias options for ascending and descending trails with grades up to 20%, shown in Figure 12 below. The weight bias data has a percentage element, so any rider mass can be used in the model.

Figure 12: Weight Bias Data

The team also changed inputs to match our new test bike, a 2020 Niner Rip 9 RDO, which has a different mass, leverage ratio, and rear shock travel. The new math model displays ideal displacement curves for each trail type when traveling at an average speed over a single average bump. These may be viewed in Appendix A. The Blue Square page can be viewed below in Figure 13.

	A	B		D			G	H			M	N	\circ	D	\circ	R	S
							Calculated Values						Average Speed	lybar	$3.58 \mid m/s$		
	Initial Values						Front Fork	Rear Spring					Average Bump Height	vbar	0.127 m		
	Bike Mass	lbm.	14.06 kg			Time(s)		Underdamped Underdamped					Average Bump Frequency	fbar	1.47 1/s		
	Person Mass	pm		95 _{kg}		0.01	0.005534739	0.005466821									
	Overall Mass	m	109.06 kg			0.02	0.010189318	0.009903441									
	Front sprung mass	mf	31.39 kg			0.03	0.014038571	0.013375464									
	Rear sprung mass	mr	77.67 kg			0.04	0.017155791	0.015960352					Blue Square Calculated Values				
8	Front fork stiffness	kF		4000 N/m		0.05	0.019612089	0.017743649	0.03								
a	Rear spring stiffness	kR	20000 N/m			0.06	0.021475887	0.018815638									
10	Front damper value	λF		500 _{kg/s}		0.07	0.022812499	0.019268433									
	Rear damper value	λR		1400 kg/s		0.08	0.023683828	0.019193516	0.025								
	Front damping coefficient cF		0.7056 Ns/m			0.09	0.024148136	0.018679685									
13	Rear damping coefficient CR		0.5616 Ns/m			0.1	0.024259896	0.017811413	0.02								
14	Front natural frequency	Fwn	11.2888 1/s			0.11	0.024069708	0.016667575									
15	Rear natural frequency	Rwn	16.0466 1/s			0.12	0.023624273	0.015320514									
16	Front Initial shock speed	lv0		$0.6 \, \mathrm{m/s}$		0.13	0.02296642	0.01383542	0.015								
17	Front sag	xOf		0 _{cm}	0.030	0.14	0.022135171	0.012269977									
18	Rear sag	xOr		0 _{cm}	0.015	0.15	0.021165842	0.010674243	SHOCK DISPLACEMENT (M)	٠							
19	Rear initial shock speed	vOr	$0.2400 \, \text{m/s}$			0.16	0.020090172	0.009090732	0.01	۰							
20	Front damped frequency	Fwd	7.9999 1/s			0.17	0.018936481	0.007554659		٠							
21	Rear damped frequency	$R\omega d$	13.2768 1/s			0.18	0.017729833	0.006094316	0.005	٠							
22	Front Fork Travel	FT	$0.15 \mbox{ m}$			0.19	0.016492227	0.004731547		٠ ٠							
23	Rear Shock Travel	RT	0.05 _m			0.2	0.015242785	0.003482299									
24	Leverage Ratio	LR		2.5 unitless		0.21	0.013997954	0.002357221									
25	Grade direction	In	Ascending	unitless		0.22	0.012771705	0.001362285		0.2			0.6 0.8				1.2
26	Grade Rating	lG	>10%			0.23	0.011575731	0.000499419									
27	Front Bias	FB	28.78 %			0.24	0.010419649	-0.000232873	-0.005				TIME (S)				
28	Rear Bias	RB	71.22 %			0.25	0.009311189	-0.00083891									
29						0.26	0.008256382	-0.001325266			• Calculated Values Front Fork Underdamped		• Calculated Values Rear Spring Underdamped				
30						0.27	0.007259741	-0.001700246									

Figure 13: Blue Square Trail Page

The new model also has a dedicated page that tells users how to set up and adjust the front fork and rear shock based on their mass and trail selection. Figure 14 below shows this page.

	A	B		D							
				Fork and Shock Setup Guide							
	Rider Weight		95 _{kg}								
			Fork Setup		Shock Setup						
4	Fork Pressure		87 psi	Shock Pressure	265 psi						
	Rebound		4 clicks from closed (all the way clockwise)	Rebound		3 clicks from closed (all the way clockwise)					
6	Compression	Open		Compression	Middle Setting						
8											
Q				Adjustments for terrain							
10	Are you going up or down?		Descending								
11	What terrain are you riding?		Blue Square								
12			Fork Adjustments	Shock Adjustments							
13	Damper Value		500 kg/s	Damper Value	1400 kg/s						
14	Rebound Adjustment	$+1$	Clicks	Rebound Adjustment +2		Clicks("-" means going towards slow, "+" is going towards fast					
15	Compression	Open		Compression	Open						
16											

Figure 14: Shock Setup Page

6.1.2 Design Iteration 2: Change in Testing Apparatus Discussion

Originally, the team planned to use a VL53L0X laser time of flight sensor attached to an Arduino Mega to calibrate and validate the math model. Images of this system are below.

Figure 15: Arduino System

Coding and practical application of the system turned out to be too tedious and clunky, so the team opted to use a readymade data acquisition system from Motion Instruments. Their XC-Enduro Pro System has linear potentiometers sized for a mountain bike fork and rear shock, with sending units that connect via Bluetooth to an iOS app. Images of this system on our test bike can be viewed below and in Appendix B.

Figure 16: Linear Potentiometer System on Bike

This system gave great data that was easy to access and analyze. It truly helped the team finish on time. The front potentiometer mounted up easily with included hardware, but due to the carbon fiber cage around the rear shock, 3D printed parts and zip ties were required to attach the rear potentiometer.

6.1.3 Design Iteration 3: Change in Device Discussion

The design originally was based on a similar platform to the current final design. These platforms have both been based around a mechanical powered system that relies on shifter wire to drive the suspension adjustments. The major changes come in the form of the way that the rear bracket, adjustment knob, and shifter plate direct the wires around the assembly. The rear bracket contains changes in clearance-based sections of the part. This allows the wire routing to not interfere with the frame as the bike moves throughout the suspension travel range. The suspension adjustment knobs received iteration changes based around how we secured and wrapped the wire around the knobs. This allows the knobs to have better wire containment and reduces the chance of the wires and assembly to interfere with other bike components. The shifter plate which is shown in the figure below adopts a tubular design that increases strength while shifting and creates a smoother shifting mechanism.

Figure 17: Shifter Assembly

The original device iteration functioned, although not reliably. Once the team employed the design changes and assembled the final iteration, our problems of clearance, routing, and fluidity were alleviated. Overall, the final device is mechanically driven and based upon the mathematical outputs, allowing a user to change shifter positions based on optimized mathematical model data.

7 RISK ANALYSIS AND MITIGATION

Due to our product targeting average everyday mountain bike riders, mitigating risk is extremely important. Thankfully, most of the risk comes from the bike itself. Compared to the bike, our device has great risk to reward ratio. The mathematical model has minimal if not zero risk to using it. While the device may have risks, they were minimized during the design process. For example, we made sure to choose reliable methods of attaching the device, such as zip ties, to the bike instead of gluing it to the bike.

7.1 Potential Failures Identified First Semester

During the first semester we were entertaining the idea of adding a device, but it was far from confirmed. Which meant that our product would have been the mathematical model alone. Because the mathematical model relies on the settings that exist already on the bike, it causes no additional risk. Which is why we did not include a risk analysis last semester.

7.2 Potential Failures Identified This Semester

As the second semester started and we confirmed that we would be able to add a device in addition to the mathematical model, our potential risks increased. The risks the device introduced ranged from "may cause injury" to failure of the device. To state the main risks that could occur from highest to lowest risk. First, the wire of the device could scratch or get stuck on the rider or any of their clothes. The device pieces falling from the bike. Lastly, the device could jam up and stop working.

7.3 Risk Mitigation

Giving how the risks accompanying the device were not that severe they were relatively easy to mitigate. Also, because we were totally familiar with what we want the device to do due to the stage of the mathematical model we started the device design in. The mitigation process started all the way at the concept generation stage. When the design team generated design concept that would not cause high risks from the start. This was then reinforced by doing a Pugh chart and taking not of concepts that achieved a negative score in safety. Our main criterion was the comfort of the rider, this meant that any concepts involving sharp objects in areas where the rider may reach was unacceptable. This, along with causing the rejection of a few other concepts involving possible risks, caused the risk requiring mitigation in the next stages to minimize. One example of a concept that was rejected involved gluing the device pieces to the bike as well as together. This was scrapped off due its low durability and higher risk of the device components falling off the bike and instead was replaced with multiple durable zip tight as well as some minor welding. As for the second risk involving the wire potentially causing harm to the rider, this was mitigated with two actions. As a start, we thought about ways we can keep the design the same but mitigate by other methods. Which is when we came up with the idea of using a thicker wire, this would increase the surface area meaning less likely hood of the wire scratching the used as well as increasing its durability and reducing the chances of it cutting off. Another action that caused the reduction of this risk was having the device solely power by the used, which means everything is controlled by the user and the wire will not move at extreme speeds that may cause harm. Having the device not electronically powers my decrease points from the rider's comfort but it also adds point in price efficiency. Lastly, the risk involving the jamming of the device was mitigated by having the parts in covered areas, areas like the inside of the shock's cage and on top of the bike's frame to cover from any mud or unwanted debris. As well as including the ease of cleaning in the design of the device.

8 ER Proofs

At the beginning of the project, the team outlined the Engineering Requirements (ERs) that would help guide and set boundaries for the project. These engineering requirements aligned with what was needed from the customer and were each given technical requirements to hit. The following section discusses the team's proof in hitting all of these targets.

8.1 ER Proof #1 – Spring and Damping Rate Critical for all Cases

In order to validate that the mathematical model was working the way it should be, the team used linear potentiometers mounted to the front and rear shocks of the test bike and a phone app to track the displacement data of the potentiometers. The following figures show the output graphs of the tests done to prove that the model is working.

The closer the two lines are collinear on the graph, the closer the shocks on the bike are to being balanced and critically damped. To test the model, the bike was tested at the correct settings as told by the mathematical model (figure 18a), then tested at the next two settings as told by the model (figure 18b and 18c). Since the lines are closest on the suggested suspension setting, it proves that the model is outputting valid suspension settings. Further testing was done on other trail types with all suspension settings.

8.2 ER Proof #2 – Validate Mathematical Model with Other Model

The proof of this ER comes with the proof of the first ER. The team's model was validated using the model created by the company that created the linear potentiometers. Since no product on the market doesn't do the same task as the team's math model, a direct comparison couldn't be done. Instead, output data from the linear potentiometers was used to make sure that the mathematical model was outputting correct settings for the bike.

8.3 ER Proof #3 – Compatible with Test Bike

Because this ER applies to both the mathematical model and the physical design, it was very important to the team that this technical target was met. By the time the team got the test bike, the mathematical model was already in a place that it could be tested with the bike. As mentioned above, the team used linear potentiometers to test the validity of the mathematical model. All of those tests were done on the test bike which proves that the mathematical model works with the test bike. As for the physical model, the team just used the test bike to model the device off of. Since the device was really just a proof of concept, the team didn't try to create a universal fit device, instead opting for something that would exclusively work on the test bike.

8.4 ER Proof #4 – Minimize Weight Addition

The ER target set for the weight was less than 45g, in order to hit close to that target, the team used lightweight 3D printed parts with minimal infill to keep the device as light as possible. The device as a whole came out just under the weight limit with the additional weight from the shifter cable and cable housing.

8.5 ER Proof #5 – Compact Design to Fit on Handlebars

Through the project, the design of the device ended up changing too much to keep the design to fit on the handlebars. In an attempt to keep the design simple and effective, the team opted to mount the device on the top tube of the bike near the handlebar stem. The ER target set was $3cm^2$ which in hindsight is incredibly small. The final device switcher ended up being closer to 60cm^2 in area in order to accommodate the slider design. Even though this difference seems large, the device still sits comfortably on the frame of the bike without taking up a lot of the rider's cockpit.

8.6 ER Proof #6 – Durability

Since the device ended up being made out of lightweight 3D printed material, the target pressure of 50 MPa ended up being a little too intense for the purposes of this project. Since the device at the end of the project was closer to a proof of concept rather than a final device, the team was more worried about the device working so not much testing was done in terms of durability. The main source of problems in terms of durability the team would have seen should have been the sliding knob that selected the different settings. The team never had a problem with that breaking so it was well within the durability limits.

9 LOOKING FORWARD

Looking forward, the team would like to see the mathematical model turned into a web based or iOSbased app to allow for easier access while riding the bike on the trail. Making the app available on devices other than a computer will help the user have an even better experience with our mathematical model. On the device side of the project, the team would continue refining the design and creating a device that can be brought to market. The current state of the device is still in the prototyping phase but does show how the concept could work. More details about the future work for the device will be discussed below.

9.1 Future Work

The team has several ideas for how to best implement components of our model and design in the future. The main iteration we would like to make with our mathematical model design process is to convert the user interface to a web or app-based interface. This would increase the usability of the model and its ability to optimize suspension settings on the trail. As for the device, the current state is developed very well. If we were to continue this design process one main change would be the manufacturing process. Certain components in the design such as the rear shock bracket functions well as a plastic 3D printed component but others such as the suspension adjustment knobs, and shifter plate would be much more reliable and stronger if a machining process could be used. These changes combined could really elevate the feasibility to bring this device to market and would be completely manageable by our team.

10 CONCLUSIONS

Throughout the course of the last two semesters, the mountain bike suspension capstone has seen multiple iterations; starting as a project tasked with creating a data base for riders to select suspension then evolving into creating a mathematical model to adjust suspension setting based on terrain with a device that allows for easier adjustments on the trail. Initially, the problem statement for the team included compiling various suspension components so riders could have a guide that would best suit their bike and style of riding. This project would have been primarily research and seemed lacking engineering depth that the team desired. Instead of the original project goal, the team pitched the idea of creating a mathematical model to help adjust suspension based on different terrain types. This new idea was stateof-the-art and has not been replicated yet. Utilizing the spring mass dashpot differential equations, weight biases based on trail grade, and spring stiffness for air shocks to name a few variables, the team created the mathematical model you have seen above. Midway through the first semester, the team was running into an issue that it is hard to work on one mathematical model with six people. To help alleviate the issue of some members not having much to do, a new component of the project was created. One issue with adjusting the rebound and compression on mountain bikes is that you must reach below your seat to make these adjustments, taking your focus off the trail. This could lead to your ride being slowed down or even crashes in worst case scenarios. To solve this problem, the team came up with the idea of mounting a device on the handlebars or towards the front of the bike that makes the adjustments on the shock without having to reach below your seat or get off the bike. Originally this device was going to be wirelessly operated, but after further consideration creating a fully mechanical device was more viable. Diving into the second semester, the team kicked the project into overdrive with a rapid design process that other teams spent the entire previous semester on. With the accelerated design process underway, the team still had to finish our mathematical model as well. The model needed a page to relate the mathematics we used to create our model to actual adjustments on the bike and everyday rider could understand. Once the team completed the model, we used a linear potentiometer from Motion Instruments and performed a series of validation testing to ensure our model outputs were optimized. With the mathematical model done and testing data proving our model outputted correct settings, the team began rapid prototyping with the device. For the final presentations, the team was able to put together a device as a proof of concept to show how our system works. To close out the final semester, we are still iterating on our initial prototype to make a more robust system. This project has taught every one of us how fluid projects can be and has shown how we are not afraid to create a project that best suits us. We have utilized all of our knowledge from our undergraduate careers to put together a project that has been fun and challenging to work on. As a team, we hope to keep learning and putting our engineering knowledge to use in the real world knowing how much we can accomplish if we put our minds to it.

10.1 Reflection

For the first half of the semester, the team used the mathematics behind most common engineering problems to pave the path for our work in the second semester. The main goal for the project was to create a mathematical model which does not affect public health, safety, or welfare, but promotes a better experience for mountain bikers. This project is not a revolutionary advancement in the human race, but sometimes engineers are not designing to save lives. Instead, we are taking a complicated system in a hobby that millions of people partake in and making this more convenient for them. A lot of times engineers are designing products with ease of use and simplicity in mind. The team was tasked with creating a mathematical model to help people understand their suspension easier. To take this one step farther, we created a device that allows for suspension adjustments on the trail without having to step off the bike. This device will allow the everyday rider to tune their suspension so that it fits each terrain type they are riding on. By not reaching below their seat to adjust suspension, mountain bike riders will be safer on the trail and ultimately faster. This device adjusts the rebound and compression settings on the rear shock of the bike, and if it fails, the suspension might not be perfectly set up. There are no real risks when the device fails and would not actually hurt the rider or other people. With this, there are no

environmental guidelines for this sort of device. We created a mathematical model to help mountain bikers adjust their suspension based on terrain types and a device to help make these adjustments easier, so we did not face strict regulations like some other teams might have.

10.2 Postmortem Analysis of Capstone

Initially, the goal statement for this project revolved around creating a database for mountain bike riders to use to help them adjust or tune their suspension. After some initial discussions, the team decided to take a slightly different route that still met the customer's needs. The new route for the project was based around creating a mathematical model to help riders adjust their suspension to various terrains. Once this idea was thought out and in motion, the team decided to create a physical device that helps riders adjust suspension on the fly with the help of our mathematical model.

10.2.1 Contributors to Project Success

For this project, the team completed our original mission; then the team went above the original expectations and created our own aspect of the project. As mentioned previously, the team took the original problem statement as a rough idea of what the project was supposed to be about. After some deliberation, the team came to the realization that we wanted to do something slightly different. So, we created the mathematical model idea to help riders adjust their suspension based on terrain. Midway through the first semester, there was a lack of tasks for some group members and the design side of this project was created. Some of the most positive aspects of the project included our development time on the device and the overall depth of the mathematical model. Since the team came up with the idea of the device later in the project, we were on an accelerated timeline compared to other groups. This proved to be a challenge but ended up getting done with a completed device manufactured. On the mathematical side of things, we took all of the necessary steps to ensure the model was comprehensive and had optimized outputs with real life testing. There were brief spells of lack of motivation for the project, but the team always came together and kicked the project into overdrive when we needed it most. Each team member has different strengths and weaknesses which contributed to the performance of the team greatly. We are all passionate about bikes and this project, so it always felt natural meeting with the team and discussing the future of the project. Occasionally the team did experience members not contributing, whether it was from lack of motivation or other reasons. Even though there were some minor instances of this, the team overall worked very hard to create a project that was exciting and innovative. Organizational tasks included separating into two separate groups where we could focus on the different aspects of the project. The team also used Microsoft Teams as a platform to hold meetings and share documents. This allowed for cohesive contributions to the project. Over the course of the last two semesters, the team has learned how engineering can be fluid and projects can change. We researched mountain bike kinematics, dynamics, and differential equations in the first part of this project. Then the team took all of our design knowledge and applied that to creating a working prototype for a system that allows for riders to adjust suspension settings on the trail. We applied knowledge of testing and experimental methods to ensure our model was valid. 3D modeling and printing was also a huge contributor to the project and helped us create our prototype. Collectively, the team applied all of our knowledge to a project we are passionate about and were able to compile it into a great senior capstone.

10.2.2 Opportunities/areas for improvement

The team concluded this year of capstone feeling accomplished that all goals set out from the start of the project were completed throughout a very detailed engineering design process. One issue the team reflected upon and realized there was room for improvement was in the timeline of the project completion. We utilized the majority of first semester to focus on the mathematical modeling and later determined that enacting a more incorporated project schedule where we took on the design process

earlier could have allowed more manufacturing time. This could have allowed the team to machine certain design components for increased strength and while this was not necessary to the function of the device, it may have allowed more practice with fundament engineering processes and further increased confidence in the long-term durability of the design. This is based around the current state of the design where CAD and 3D printing was essential to the construction of our device. Overall, the team created a detailed and well-designed mathematical model and physical device that exceeded expectations of our clients and original goals. The project allowed the team to develop many new and refined engineering skills while also contributing to skills of teamwork, client communication, and project management.

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12 APPENDICES

Calculated Values Front Fork Underdampe

12.1 Appendix A: Math Model Outputs

